ID:____________________

The Scots College

2007
Trial
Higher School Certificate
Examination

Mathematics

General Instructions

- **Reading time: 5 minutes**
- **Working time: 3 hours**
- **Write using blue or black pen. (sketches in pencil)**
- **Board approved calculators may be used.**
- **A table of standard integrals is provided at the back of this paper.**
- **All necessary working should be shown in every question.**
- **Answer each question on a separate booklet.**
- **Total marks 120**
- **Attempt questions 1-10**
- **All questions are of equal value.**

Students are advised that this is a Trial examination **only** and cannot in any way guarantee the content or the format of the Higher School Certificate examination.

Total Marks 120
Attempt Questions 1 – 10
All questions are of equal value
Answer each question in a separate booklet.

Question 1 (12 marks) Use a separate booklet

(a) The volume of a sphere is $78\ cm^3$. Find the diameter of the sphere to 3 significant figures using the formula $V = \frac{4}{3}\pi r^3$ [2]

(b) The roots of an equation are -7 and $\frac{4}{3}$. Write its equation in the form $ax^2 + bx + c = 0$ where a, b and c are integers. [2]

(c) Solve the equation $(a-5)^2 - 1 = (a-4)(a+3)$. [2]

(d) Differentiate $\frac{1}{4}x^4 - \frac{1}{3}\sqrt{x^3}$ [2]

(e) Find the values of x for which $|3x - 7| > 2$ [2]

(f) Express $0.\dot{4}0\dot{3}$ as a simple fraction. [2]

Question 2 (12 marks) Use a separate booklet

(a) The mid point of the line joining the points A and B is $(-6,7)$. If A is the point $(2,5)$ find the coordinates of B. [2]

(b) Solve the pair of simultaneous equations: [3]
$2x - y - 10 = 0$ and $y = x^2 - 3x - 10$

(c) For the points $A(-4,3)$, $B(3,-1)$ and $C(-3,-4)$ find:

(i) the gradient of the line through A and C. [1]

(ii) the exact distance between B and C. [1]

(iii) the mid point of the interval AB. [1]

(iv) the equation of the line parallel to BC and passing through A. [2]

(v) the equation of the line perpendicular to AC and passing through B [2]

Question 3 (12 marks) Use a separate booklet

(a) Solve $\sin\theta = \dfrac{\sqrt{3}}{2}$ for $0 \le \theta \le 2\pi$. [2]

(b) Differentiate with respect to x:

(i) $x^2 \cos x$ [2]

(ii) $\dfrac{x}{4-x^2}$ [2]

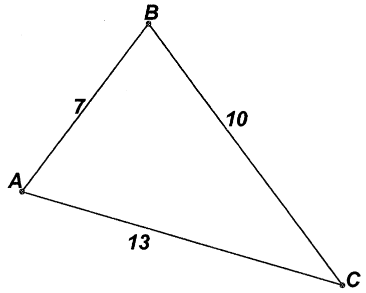
(c) (i) Find $\displaystyle\int \frac{4x}{x^2+7}\,dx$ [2]

(ii) Evaluate $\displaystyle\int_0^{\pi/8} \cos 4x \; dx$ [2]

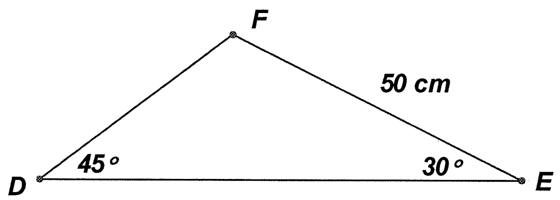
(d) Find the equation of the tangent to the curve $y = e^{5x}$ at the point $(0,1)$. [2]

Question 4 **(12 marks) Use a separate booklet**

(a) Find the size of the largest angle in the triangle below (not to scale), correct to the nearest minute. [2]



(b) In the triangle DEF below :



(i) Show that the length of $DF = 25\sqrt{2}$ [2]

(ii) Find the area of the triangle DEF, correct to 2 decimal places. [1]

(c) Consider the function $f(x) = x^3 - 6x^2 - 8$

(i) Find the coordinates of the stationary points and determine their nature. [4]

(ii) Find the coordinates of the point of inflection. [2]

(iii) Sketch the graph of the function, showing all the above information. [1]

Question 5 (12 marks) Use a separate booklet

(a) Joseph's first year salary is \$42000. If he stays with the same company, his annual salary will increase by 4.5% each year.

(i) What will Joseph's annual salary be in the 15^{th} year (correct to the nearest dollar)? [2]

(ii) In what year will his annual salary first exceed \$65000? [2]

(iii) What will be his total earnings in his 35 years of employment in the company? [2]

(b) Rachel has a viral infection and the virus count per ml of blood sampled is 109296. She is treated with antibiotics and each day the number of virus is reduced by 2450 from the previous day.

(i) How many virus will Rachel have at the end of the 14^{th} day? [2]

(ii) By which day will she have got rid of half the virus? (correct to the nearest day). [2]

(iii) At the end of which day will she be completely free from the virus? (correct to the nearest day). [2]

Question 6 (12 marks) Use a separate booklet

(a) Solve the equation for p, $9^p - 12\times 3^p + 27 = 0$ [3]

(b) Let α and β be the roots of the equation $3x^2 - 6x - 1 = 0$. Find : [4]

(i) $\alpha + \beta$

(ii) $\alpha\beta$

(iii) $\alpha^2\beta + \alpha\beta^2$

Question 6 continued...... [5]

(c) For what values of p does the equation $5x^2 - (6p + 4)x + 5p = 0$ have :

(i) Equal roots?

(ii) One of the roots equal to 3?

Question 7 (12 marks) Use a separate booklet

(a) Evaluate $\sum_{n=4}^{7}(6n + 5)$ [1]

(b) (i) Sketch a neat graph of $y = 2\sin 2x$, for $0 \le x \le 2\pi$. [2]

(ii) Copy and complete the following table of values for $y = \frac{2}{5}x$ (correct to one decimal place where necessary). [2]

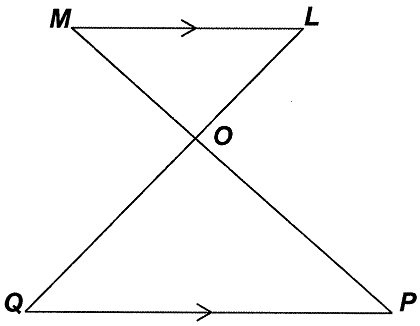
x	0	π	2π
y			

(iii) Sketch the graph of $y = \frac{2}{5}x$ on the same diagram as part (i). [1]

(iv) Hence determine the number of solutions of the equation $10\sin 2x - 2x = 0$ for $0 \le x \le 2\pi$. [1]

Question 7 continued...

(c)



The lines LQ and MP intersect at O as shown in the figure above. $ML \parallel QP$, LM is $5m$, LO is $3.5m$, MO is $4m$ and OQ is $8m$.

(i) Copy the above diagram in your exam booklet and mark on it all the given information. [1]

(ii) Prove that the triangles MLO and OPQ are similar. [3]

(iii) Find the length of PQ, correct to two decimal places. [1]

Question 8 (12 marks) Use a separate booklet

(a) Roger took a jug which initially had 300 ml of water in it and left it out in the rain. He waited for 30 seconds until it was full. During this time the volume flow Rate R of water, in milliliters per second, into the jug was given by $R = 3(50 - t)$.

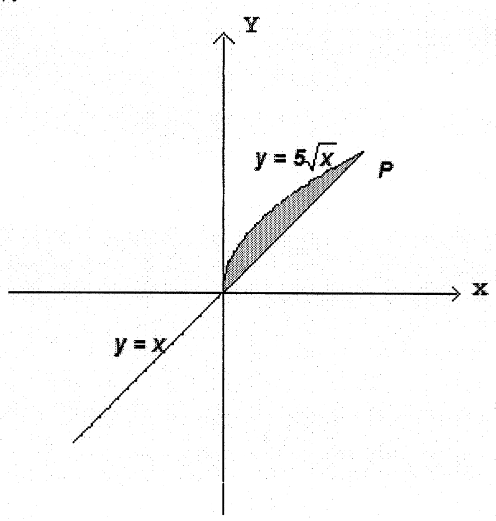
(i) Find the formula for the volume V of water in the jug after t seconds where $t \leq 30$. [1]

(ii) Find the total capacity of the jug. [2]

(iii) How long did it take before the bottle was half full? (correct to one decimal place) [3]

Question 8 continued......

(b) The region enclosed by the curve $y = 5\sqrt{x}$ and $y = x$ is rotated about the $x-axis$ as shown in the figure below.



(i) Find the coordinates of P. [1]

(ii) Find the volume of the solid of revolution around the x-axis. [5]

Question 9 (12 marks) Use a separate booklet

(a) Find the value of $\log_8 512$. [1]

(b) There is a 65% chance that the sport training on Tuesday will be cancelled due to rain, and a 75% chance that training on Thursday will be cancelled due to rain.

Draw a probability tree diagram and find the probability that : [2]

(i) Training is cancelled on Tuesday but held on Thursday. [1]

(ii) Training will be cancelled on **only** one of these days. [1]

(iii) Training will be cancelled on both these days. [1]

Question 9 continued...

(c) Find an approximation to the definite integral $\int_0^2 x^3\,dx$, using ***two*** applications of the trapezoidal rule. [3]

(d) (i) Differentiate $y = \dfrac{1}{\cos x}$ [1]

(ii) Hence, or otherwise, evaluate $\displaystyle\int_0^{\pi/4} \tan x \sec x\,dx$ [2]

Question 10 (12 marks) Use a separate booklet

(a) Equation of a parabola is $x^2 = 12y + 6x + 15$

By completing the square, express the equation in the form $(x-h)^2 = 4a(y-k)$ and find :

(i) the coordinates of the vertex [2]

(ii) the coordinates of the focus [1]

(iii) the equation of the directrix [1]

(b) Find the derivative of $y = \log_e(5x^3 + 2x)$ [2]

(c) (i) Show that $\dfrac{1-2x}{1+x}$ can be written as $-2 + \dfrac{3}{1+x}$. [2]

(ii) Hence evaluate $\displaystyle\int_2^e \frac{1-2x}{1+x}\,dx$. [2]

(d) Find the area of a sector of a circle with radius $7\sqrt{2}$ meters if the angle at the centre is 70^0, correct to two decimal places. [2]

The End

Standard Integrals

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; x \neq 0, \text{if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

HSC 2 Unit Trial

Q.1.

(a)
$$78 = \frac{4}{3}\pi r^3$$
$$r^3 = \frac{78 \times 3}{4\pi}$$
$$r = \sqrt[3]{\frac{78\times 3}{4\pi}} \quad ✓$$
$$r = 2.6505$$
$$r = 2.65 \quad ✓$$

(b)
$$(x+7)(x-\tfrac{4}{3}) = 0 \quad ✓$$
$$x^2 - \tfrac{4}{3}x + 7x - \tfrac{28}{3} = 0$$
$$3x^2 - 4x + 21x - 28 = 0$$
$$3x^2 + 17x - 28 = 0 \quad ✓$$

(c)
$$(a-5)^2 - 1 = (a-4)(a+3)$$
$$\cancel{a^2} - 10a + 25 - 1 = \cancel{a^2} + 3a - 4a - 12 \quad ✓$$
$$36 = 9a$$
$$a = 4 \quad ✓$$

(d)
$$y = \frac{1}{4}x^4 - \frac{1}{3}x^{3/2}$$
$$\frac{dy}{dx} = \frac{4}{4}x^3 - \frac{3}{2}\times\frac{1}{3}x^{1/2} \quad ✓$$
$$= x^3 - \frac{\sqrt{x}}{2} \quad ✓$$

(e)
$$+(3x-7) < -2 \quad \text{or} \quad 3x - 7 > 2$$
$$3x - 7 < -2 \qquad 3x > 9$$
$$3x < 5 \qquad x > 3 \quad ✓$$
$$x < \frac{5}{3} \quad ✓$$

(f)
$$\text{Let } x = 0.\dot{4}0\dot{3} \quad (1)$$
$$1000x = 403.\dot{4}0\dot{3} \quad (2) \quad ✓$$
$$(2)-(1) \Rightarrow \quad 999x = 403$$
$$x = \frac{403}{999} \quad ✓$$

Q. 2.

(a)

Let B be (x_2, y_2)

$$\frac{2 + x_2}{2} = -6 \quad \text{and} \quad \frac{5 + y_2}{2} = 7$$

$$2 + x_2 = -12 \qquad 5 + y_2 = 14$$

$$x_2 = -14 \qquad y_2 = 9$$

$\therefore$ $B(-14, 9)$

(b) $2x - y - 10 = 0$ and $y = x^2 - 3x - 10$ — (II)

$\Rightarrow 2x - 10 = y$ — (I)

(I) = (II) $\Rightarrow$

$$2x - 10 = x^2 - 3x - 10$$

$$0 = x^2 - 5x$$

$$x^2 - 5x = 0$$

$$x(x-5) = 0$$

$$x = 0 \quad \text{OR} \quad x = 5$$

$x = 0$ $y = 2\times 0 - 10$ OR $x = 5$ $y = 2\times 5 - 10$

$y = -10$ $\qquad = 0$

$(0, -10)$ $\qquad (5, 0)$.

Q.2. cont.....

(c) $A(-4,3)$, $B(3,-1)$ and $C(-3,-4)$

(i) $m_{AC} = \dfrac{-4-3}{-3+4} = \dfrac{-7}{1} = -7$ ✓

(ii) $d_{BC} = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$= \sqrt{(-3-3)^2+(-4--1)^2}$

$= \sqrt{(-6)^2+(-3)^2}$

$= \sqrt{36+9} = \sqrt{45}$ ✓

or $= 3\sqrt{5}$

(iii) $M_{AB}\left(\dfrac{x_1+x_2}{2}, \dfrac{y_1+y_2}{2}\right)$

$M\left(\dfrac{-4+3}{2}, \dfrac{3-1}{2}\right)$

$M\left(-\frac{1}{2}, 1\right)$ ✓

(iv) Gradient of BC, $m_{BC} = \dfrac{-4--1}{-3-3} = \dfrac{-3}{-6}$

$= \frac{1}{2}$.

slope of parallel line $= \frac{1}{2}$ & point $A(-4,3)$.

equation $y-3 = \frac{1}{2}(x--4)$ ✓

$y-3 = \frac{1}{2}(x+4)$

$2y-6 = x+4$.

$x-2y+10 = 0$ ✓

(v) slope of line perpendicular to AC $= \frac{1}{7}$ & $B(3,-1)$

equation : $y--1 = \frac{1}{7}(x-3)$ ✓

$7y+7 = x-3$

$x-7y-10 = 0$ ✓

Q.3.

(a) $\sin\theta = \frac{\sqrt{3}}{2}$

$\theta = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$\theta = \frac{\pi}{3}, \frac{2\pi}{3}$

(b)(i) $y = x^2 \cos x$ $\qquad u = x^2 \quad v = \cos x$.

$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$.

$= x^2(-\sin x) + \cos x\,(2x)$ ✓

$= -x^2 \sin x + 2x\cos x$. ✓

(ii) $y = \frac{x}{4-x^2}$

$\frac{dy}{dx} = \frac{v\,du/dx - u\,dv/dx}{v^2}$

$= \frac{(4-x^2)1 - x(-2x)}{(4-x^2)^2}$ ✓

$= \frac{4-x^2+2x^2}{(4-x^2)^2}$

$= \frac{4+x^2}{(4-x^2)^2}$ ✓

(c) (i) $\int \frac{4x}{x^2+7}\,dx$ $\qquad$ Let $u = x^2+7$

$\frac{du}{dx} = 2x \Rightarrow du = 2x\,dx$

Now $2\int \frac{2x}{x^2+7}\,dx$. ✓ $\qquad \cancel{\frac{du}{2x} = dx}$.

$2\int \frac{1}{u}\,du$

$2\log u + C$

$2\log(x^2+7) + C$ ✓ $\qquad \left(-\frac{1}{2} \text{ if no } C\right)$

Q.3. cont....

(c) (ii) $\int_0^{\pi/8} \cos 4x \, dx$

$\left[\frac{1}{4} \sin 4x\right]_0^{\pi/8}$ ✓

$\left[\frac{1}{4} \sin 4(\pi/8) - \frac{1}{4} \sin 0\right]$

$\Rightarrow \frac{1}{4} \sin \frac{\pi}{2} - 0$

$\Rightarrow \frac{1}{4}$ ✓

(d) $y = e^{5x}$ at the point $(0,1)$

gradient $= \frac{dy}{dx} = 5e^{5x}$

at $x = 0$, $\frac{dy}{dx} = 5e^{5\times 0} = 5$ ✓

equation of tangent

$y - 1 = 5(x - 0)$

$y - 1 = 5x$

$5x - y + 1 = 0$ ✓

Q.4.

(a) $\angle B$ is the largest

and $\cos B = \dfrac{a^2+c^2-b^2}{2ac}$

$= \dfrac{10^2+7^2-13^2}{2\times10\times7}$ ✓

$= \dfrac{-20}{140}$

$\cos B = -0.1429$ (obtuse angle)

$B = \cos^{-1}(-0.1429)$

$= 98^\circ\,12'\,48''$

$\approx 98^\circ 13'$ ✓

(b) (i) $\dfrac{DF}{\sin 30^\circ} = \dfrac{50}{\sin 45^\circ}$ ✓

$DF = 50 \times \dfrac{\sin 30^\circ}{\sin 45^\circ}$

$= 50 \times \dfrac{1/2}{1/\sqrt{2}} = 25\sqrt{2}$ ✓

(ii) Area $= \frac{1}{2} DF \times FE \times \sin F$

$= \frac{1}{2}\, 25\sqrt{2} \times 50 \times \sin 15^\circ$ ✓

$= 228.7658$ cm^2

≈ 228.77 ✓

c (i) $f(x) = x^3 - 6x^2 - 8$

$f'(x) = 3x^2 - 12x$

$f'(x) = 0 \Rightarrow 3x^2 - 12x = 0$

$3x(x-4) = 0$

$x = 0$ OR $x = 4$

Stationary points are

$x = 0$, $y = 0^3 - 6(0^2) - 8$

$y = -8$

$(0, -8)$ and $(4, -40)$

Q.4 cont....

(c) (ii) point of inflexion $\frac{d^2y}{dx^2} = 0$

$$f''(x) = 6x - 12$$ ✓

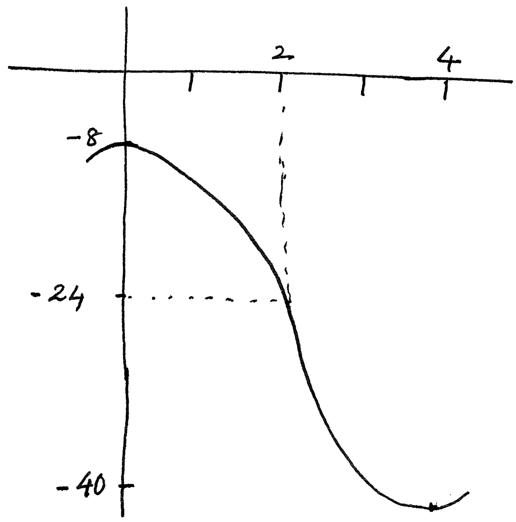
$$6x - 12 = 0$$

$$6x = 12$$

$$x = 2$$

$x = 2, \quad y = -24$ $\quad (2, -24)$ ✓

(iii)



✓

(iv) $x < 0$ and $x > 4$ ✓

Q.5.

(a) (i) $T_n = ar^{n-1}$

$T_{15} = 42000\,(1.045)^{14}$ ✓

$= 77781.69$

≈ 77782 ✓

(ii) $42000\,(1.045)^{n-1} = 65000$

$(1.045)^{n-1} = \frac{65}{42}$

$(n-1)\log(1.045) = \log\left(\frac{65}{42}\right)$ ✓

$n-1 = \frac{\log(65/42)}{\log(1.045)}$

$n-1 = 9.92$

$n = 10.92$

11^{th} year. ✓

(iii) $S_n = \frac{a(r^n-1)}{r-1}$

$S_{35} = \frac{42000\,(1.045^{35}-1)}{1.045-1}$

$= 3{,}422{,}857.96$

(b) (i) $T_1 = 109296 = a$

$T_n = a + (n-1)d$.

$T_{14} = 109296 + (13)(-2450)$

$= 77{,}446$

(ii) $109296 + (n-1)(-2450) = \frac{109296}{2}$ ✓

$(n-1)(-2450) = 54648 - 109296$

$(n-1) = \frac{54648}{2450}$

$n-1 = 22.31$

$n = 23.31$ ✓

(iii) $n-1 = \frac{109296}{2450}$ ✓

$n-1 = 44.6$

$n = 45.6$ days. ✓

Q.6.

(a) $9^p - 12 \times 3^p + 27 = 0$

$3^{2p} - 12\,3^p + 27 = 0$ ✓

Let $x = 3^p$. ✓

$\therefore$ $x^2 - 12x + 27 = 0$

$x^2 - 3x - 9x + 27 = 0$

$x(x-3) - 9(x-3) = 0$

$(x-3)(x-9) = 0$

$x = 3$ or $x = 9$.

$\Rightarrow$ $3^p = 3$ or $3^p = 9$

$p = 1$ or $p = 2$. ✓

(b) . $3x^2 - 6x - 1 = 0$

(i) $\alpha + \beta = -\frac{b}{a}$

$= -\frac{-6}{3}$

$\alpha + \beta = 2$ ✓

(ii) $\alpha\beta = \frac{c}{a}$

$\alpha\beta = \frac{-1}{3}$ ✓

(iii) $\alpha^2\beta + \alpha\beta^2 = \alpha\beta(\alpha+\beta)$ ✓

$= -\frac{1}{3}(2)$

$= -\frac{2}{3}$ ✓

C. (i) Equal roots $b^2 - 4ac = 0$

Q.6.

(c) $5x^2 - (6p+4)x + 5p = 0$

(i) $\Delta = 0$

$[-(6p+4)]^2 - 4(5)(5p) = 0$ ✓

$36p^2 + 48p + 16 - 100p = 0$

$36p^2 - 52p + 16 = 0$

$9p^2 - 13p + 4 = 0$ ✓

$(9p-4)(p-1) = 0$

$\therefore p = \frac{4}{9}$ OR $p = 1$ ✓

(ii) $5(3)^2 - (6p+4)(3) + 5p = 0$ ✓

$45 - 18p - 12 + 5p = 0$

$33 - 13p = 0$

$13p = 33$

$p = \frac{33}{13}$ ✓

Q. 7

(a) $\sum_{n=4}^{7} (6n+5.)$

$T_4 = 6\times4+5 = 29$

$T_5 = 6\times5+5 = 35$

$T_6 = 6\times6+5 = 41$

$T_7 = 6\times7+5 = 47$

152.

2nd Method :

$T_1 = 6\times1+5 = 11$

$T_2 = 6\times2+5 = 17$

$a = 11 \qquad d = 6$

$S_7 = \frac{7}{2}[2\times11+(7-1)6]$

$= \frac{7}{2}[58]$

$= 203$

$S_3 = \frac{3}{2}[2\times11+(3-1)6]$

$= 51$

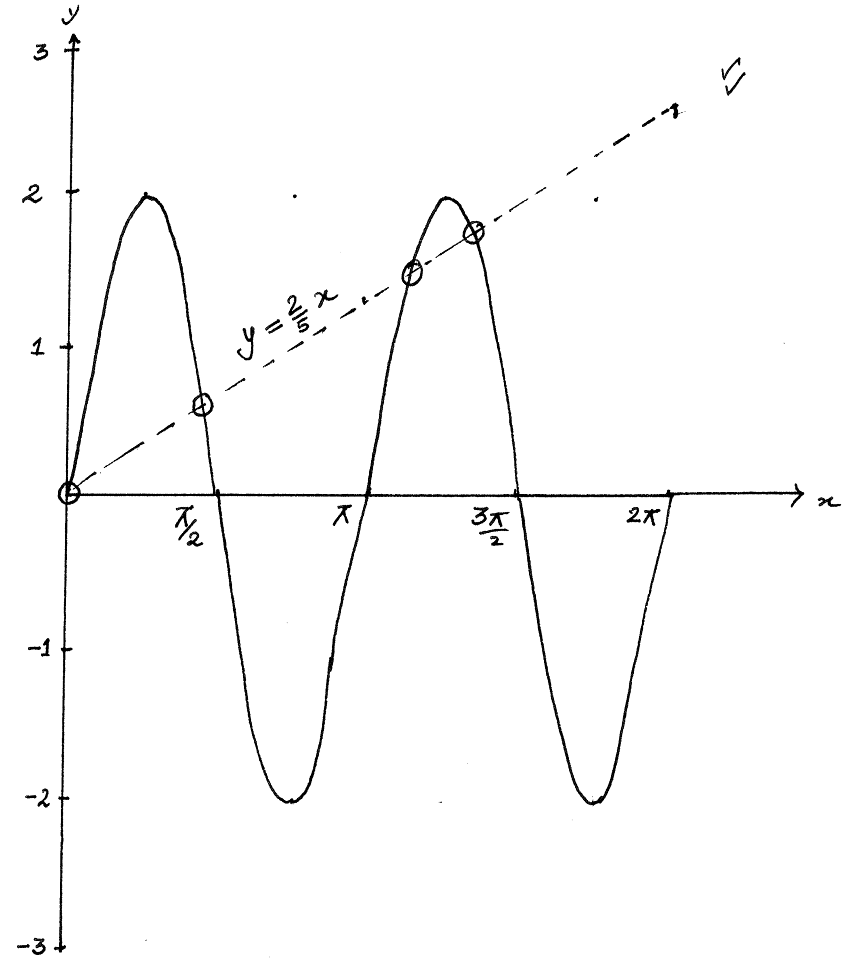
$S_7 - S_3 = 203-51$

$= 152$

Q.7.

(b)

(i)



(1)

(ii)

x	0	π	2π
y	0	1.26	2.51

(iv)

$$10 \sin 2x - 2x = 0$$

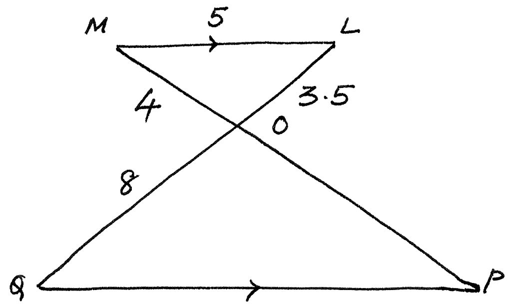
$$10 \sin 2x = 2x$$

$$2 \sin 2x = \frac{2}{5}x.$$

There are 4 solutions as shown in the graph above.

Q.7

(b) (i)



(ii) Consider $\triangle MLO$ and $\triangle OPQ$

$\angle MLQ = \angle LQP$ (alt. angles on parallel lines)

$\angle LMP = \angle MPQ$ (")

$\angle MOL = \angle QOP$ (vertically opp. angles)

$\therefore$ $\triangle MLO \,|||\, \triangle OPQ$ (AAA)

(iii) Since $\triangle MLO \,|||\, \triangle OPQ$ the sides must be in ratio.

i.e. $\frac{LO}{OQ} = \frac{ML}{QP}$

$\frac{3.5}{8} = \frac{5}{QP}$

$\Rightarrow QP = \frac{5 \times 8}{3.5}$

$= 11.4285$

$= 11.43$

Q. 8

(a) (i) $V = \int R \, dt$

$= \int 3(50 - t)\, dt$

$= \int (150 - 3t)\, dt$

$= 150t - \frac{3t^2}{2} + c$

$300 = 150(0) - \frac{3(0)^2}{2} + c$ since $V = 300$ when $t = 0$

$\Rightarrow c = 300$

$\therefore V = 150t - \frac{3}{2}t^2 + 300$ ✓

(ii) at $t = 30$ the jug was full

$V = 150(30) - \frac{3}{2}(30)^2 + 300$ ✓

$V = 3450$ mL ✓

(iii) half full $V = 1725$

$1725 = 150t - \frac{3}{2}t^2 + 300$

$\frac{3}{2}t^2 - 150t + 1425 = 0$

$t = \dfrac{-(-150) \pm \sqrt{(-150)^2 - 4 \times \frac{3}{2} \times 1425}}{2 \times \frac{3}{2}}$ ✓

$= \dfrac{150 \pm 118.11}{3}$

$= 10.6$ or 89.37. ✓

$t = 10.6$ seconds since $t \le 30$. ✓

Q. 8 (b)

$y = 5\sqrt{x}$ and $y = x$

(i) Point of intersection P

$5\sqrt{x} = x$

$25x = x^2$

$x^2 - 25x = 0$

$x(x-25) = 0$

$x = 0$ Or $x = 25$

$x = 25$, $y = 5\sqrt{25}$
$= 25$ $(25, 25)$ ✓

(ii) $V = \pi \int_a^b r^2\,dx$ ✓

First curve.

$$V = \pi \int_0^{25} (5\sqrt{x})^2\,dx$$

$$= \pi \int_0^{25} 25x\,dx$$

$$= \pi \left[\frac{25x^2}{2} \right]_0^{25}$$ ✓

$$= \pi \times \frac{25 \times 25^2}{2}$$ $= 24543.69$ cubic units. ✓

2nd curve.

$$V = \pi \int_0^{25} (x)^2\,dx$$

$$= \pi \left[\frac{x^3}{3} \right]_0^{25}$$ ✓

$$= \frac{\pi\, 25^3}{3}$$ $= 16,362.46$ cubic units

Volume of solid of revolution is ✓

$24543.69 - 16362.46 = 8,181.23$ cubic units.

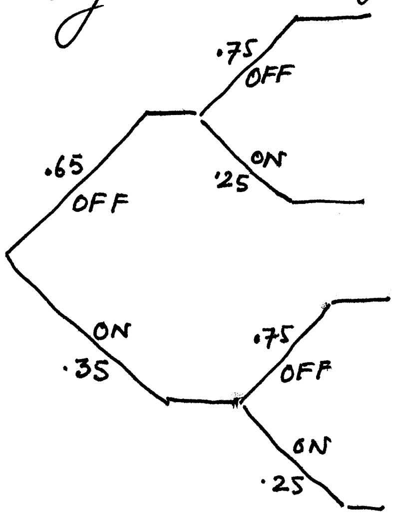
Q. 9 (a)

$$\log_8 512 = \log_8 8^3$$
$$= 3\log_8 8$$
$$= 3 \times 1$$
$$= 3 \quad \checkmark$$

OR.

$$x = \log_8 512$$
$$\Rightarrow 8^x = 512$$
$$8^x = 8^3$$
$$x = 3 \quad \checkmark$$

(b) Tuesday Thursday.



Tu Th.

OFF OFF
$.65 \times .75 = 0.4875$

OFF ON
$.65 \times .25 = 0.1625$ ✓✓

ON OFF
$0.35 \times 0.75 = 0.2625$

ON ON
$0.35 \times 0.25 = 0.0875$

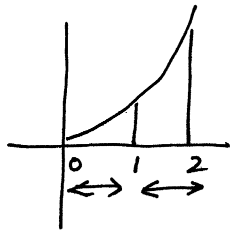
(i) $P(\text{OFF ON}) = 0.1625$ ✓

(ii) $P(\text{OFF on one day}) = 0.1625 + 0.2625$
$= 0.4250$ ✓

(iii) $P(\text{OFF OFF}) = 0.4875$ ✓

Q.9.

(c) Two applications of Trapezoidal rule.



$$A = \frac{1}{2}(1-0)\{f(0)+f(1)\} + \frac{1}{2}(2-1)\{f(1)+f(2)\}$$

$$= \frac{1}{2}\{0^3+1^3\} + \frac{1}{2}\{1^3+2^3\}$$

$$= \frac{1}{2} + \frac{9}{2}$$

$$= \frac{10}{2}$$

$$= 5 \text{ sq. units}$$

(d) (i) $y = \frac{1}{\cos x} = (\cos x)^{-1}$

$y = u^{-1}$ $\quad u = \cos x$

$\frac{du}{dx} = -\sin x$

$$\frac{dy}{du} = -1\,u^{-2} = -\frac{1}{\cos^2 x}$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$= -\frac{1}{\cos^2 x} \times (-\sin x) = \frac{\sin x}{\cos x \cos x}$$

$$= \tan x \sec x$$

(ii) $\int_0^{\pi/4} \tan x \sec x \, dx = \left[\frac{1}{\cos x}\right]_0^{\pi/4}$

$$= \left[\frac{1}{\cos \pi/4} - \frac{1}{\cos 0}\right]$$

$$= \frac{1}{\sqrt{2}/2} - \frac{1}{1}$$

$$= \frac{2}{\sqrt{2}} - \frac{1}{1}$$

$$= \sqrt{2} - 1$$

Q. 10.

(a) $x^2 = 12y + 6x + 15$

$x^2 - 6x = 12y + 15$

$x^2 - 6x + 3^2 = 12y + 15 + 3^2$

$(x-3)^2 = 12y + 24$

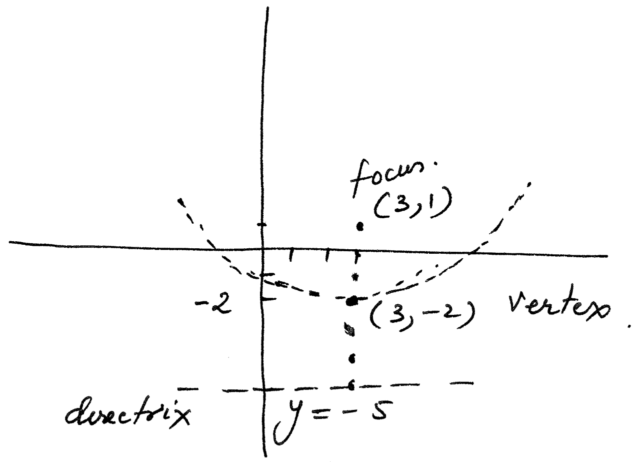
$(x-3)^2 = 12(y+2)$

$(x-3)^2 = 4 \times 3 (y+2)$ ✓

(i) Vertex $= (3, -2)$ ✓

(ii) $(3, 1)$ ✓

(iii) $y = -5$ ✓



(b) $y = \log_e(5x^3 + 2x)$

$u = 5x^3 + 2x$

$\frac{du}{dx} = 15x^2 + 2$

$y = \log u$

$\frac{dy}{du} = \frac{1}{u}$.

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

$= \frac{1}{u} \times (15x^2 + 2)$ ↑✓

$= \frac{1}{5x^3 + 2x} (15x^2 + 2)$

$= \frac{15x^2 + 2}{5x^3 + 2x}$ ✓

Q. 10

(c) (i) $\dfrac{1-2x}{1+x} = \dfrac{-2x+1}{x+1}$

$$\begin{array}{r}
-2 \\
x+1 \overline{) -2x+1} \\
-2x-2 \\
\underline{+ \quad +} \\
3
\end{array}$$

$\therefore \dfrac{1-2x}{1+x} = -2 + \dfrac{3}{x+1}$

OR.

$-2 + \dfrac{3}{1+x}$

$\dfrac{-2(1+x)+3}{1+x}$

$\dfrac{-2-2x+3}{1+x}$

$\dfrac{1-2x}{1+x}$

(ii) $\displaystyle\int_2^e \frac{1-2x}{1+x}\,dx = \int_2^e \left(-2 + \frac{3}{1+x}\right) dx$

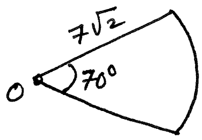
$= \left[-2x + 3\ln(1+x)\right]_2^e$

$= \left(-2e + 3\ln(1+e)\right) - \left(-4 + 3\ln 3\right)$

$= -2e + 3\ln(1+e) + 4 - 3\ln 3$

$= -0.79$

(d)



$A = \dfrac{70}{360} \times \pi (7\sqrt{2})^2$

$= 59.86\ m^2$